

The WZ Term of the Spinning String and its On-shell Structure

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Abstract

The Wess-Zumino term of the spinning string is constructed in terms of their anomalies using an extended field-antifield formalism. A new feature appears from a fact that the non-anomalous transformations do not form a sub-group. The algebra of the extended variables closes only using the equations of motion derived from the WZ term.

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1 Introduction

In a recent paper[1] we have analyzed the form of the Wess-Zumino (WZ) term at one loop for general anomalous on-shell gauge theories using an extended field-antifield formalism. The gauge degrees of freedom that become propagating at quantum level are introduced as dynamical fields. The antifield independent part of the WZ term, in the classical basis, has the usual form [2][3][4] and is expressed in terms of the anomalies and the finite gauge transformations. The full WZ term is obtained by the cohomological reconstruction procedure [5] [6].

In this paper we analyze the system of spinning string [7][8] as an example of anomalous gauge theories with open algebras. The Weyl and super-Weyl transformations become anomalous in regularizations in which the diffeomorphism invariance is preserved. The Liouville and the super-Liouville fields are introduced as extra degrees of freedom in a natural way and are propagating at quantum level. The WZ term is constructed from expressions of the anomaly and the finite anomalous transformations.

The on-shell structure of the extended formalism has a new phenomena that the algebra of two SUSY transformations of the super Liouville field does not close at classical level. Therefore we cannot obtain a solution of the classical master equation in the extended formalism at this level. However the non-closure term vanishes on-shell of its equations of motion obtained from the WZ term describing dynamical properties of the super Liouville fields. When we consider the quantum action including the WZ term and perform a canonical scale transformation depending on \hbar we can isolate terms independent of \hbar as the solution of the classical master equation. The residual term is a quantum correction and is proportional to $\sqrt{\hbar}$. It is the background term and cancels the anomaly at one loop.

This paper is organized as follows. In section 2, we briefly outline the proper solution of the classical master equation in the space of local functional of fields and antifields. In section 3 the WZ action is constructed from the super-Weyl anomaly using the form of finite transformations. Section 4 is devoted to discussion of the extended field-antifield formalism for the spinning string. Summary and discussion are given in Section 5.

2 Classical symmetry and proper solution of classical master equation

In this section we will review the classical gauge symmetries of the spinning string and the construction of solution of classical master equation.

The classical action of spinning string is given by [7][8]¹

$$S_0 = - \int d^2x \, e \left[\frac{1}{2} (g^{\alpha\beta} \partial_\alpha X \partial_\beta X - i \bar{\psi} \rho^\alpha \nabla_\alpha \psi) + \bar{\chi}_\alpha \rho^\beta \rho^\alpha \psi \partial_\beta X + \frac{1}{4} (\bar{\psi} \psi) (\bar{\chi}_\alpha \rho^\beta \rho^\alpha \chi_\beta) \right], \quad (1)$$

where X^μ and ψ^μ ($\mu=0,\dots,D-1$) are, respectively, the bosonic and fermionic string variables and we suppress the space-time indices μ . The zweibein field and gravitino are denoted by e_α^a and χ_α , respectively. The covariant derivative ∇_α is defined by

$$\nabla_\alpha \equiv \partial_\alpha + \frac{1}{2} \varpi_\alpha \rho_5, \quad (2)$$

where ϖ_α is the spin connection defined by²

$$\varpi_\alpha = \varpi_\alpha^{(0)} + \varpi_\alpha^{(1)}; \quad \varpi_\alpha^{(0)} = \frac{1}{e} e_\alpha^a \epsilon^{\beta\gamma} \partial_\beta e_{\gamma a}, \quad \varpi_\alpha^{(1)} = 2i \bar{\chi}_\alpha \rho_5 \rho^\beta \chi_\beta. \quad (3)$$

The action is invariant under the world-sheet reparametrization, local Lorentz rotation, local supersymmetry(SUSY), Weyl and super-Weyl transformations. Since these transformations are independent the algebra is irreducible. In contrast with the bosonic string the algebra is open since the commutator of two SUSY transformations of the fermion ψ closes only on-shell of the classical equation of motion $S_{0,\psi} \approx 0$. That is

$$[\delta_{SUSY}, \delta'_{SUSY}] \psi = \delta'' \psi - \frac{1}{e} \epsilon' (S_{0,\psi} \epsilon) + \frac{1}{e} \epsilon (S_{0,\psi} \epsilon'), \quad (4)$$

where ϵ and ϵ' are the parameters of the local super-symmetry transformations.

$$S_{0,\psi} \equiv \frac{\delta^r S_0}{\delta \psi} = e \overline{\left\{ i \rho^\alpha \nabla_\alpha \psi - \rho^\alpha \rho^\beta \chi_\alpha (\partial_\beta X - \bar{\psi} \chi_\beta) \right\}} \quad (5)$$

are the equations of motion of the fermionic matter. The first term of r.h.s. of (4), $\delta'' \psi$, represents a sum of reparametrization, local Lorentz and local super-symmetry transformations.

The classical algebraic structures are nicely formulated as the classical master equation (CME) in the field antifield formalism [10][11]³. It is expressed in terms of anti-bracket as

$$(S, S) = 0. \quad (6)$$

The solution of the CME for the spinning string is [15].

$$S = S_0 + X^* (\partial_\alpha X C^\alpha + \bar{\psi} \omega)$$

¹We follow the notation of [9].

² $\epsilon^{01} = -\epsilon_{01} = 1$ both for the world and local tensors $\epsilon^{\alpha\beta}$ and ϵ^{ab} .

³For reviews see [12][13][14]

$$\begin{aligned}
& + \psi^* (\partial_\alpha \psi C^\alpha + \frac{1}{2} \rho_5 \psi C_L - \frac{1}{4} \psi C_W - i \rho^\alpha \omega (\partial_\alpha X - \bar{\psi} \chi_\alpha)) \\
& + e_a^{\alpha*} (\partial_\beta e_\alpha^a C^\beta + \epsilon_b^a e_\alpha^b C_L + \frac{1}{2} e_\alpha^a C_W + 2i \bar{\chi}_\alpha \rho^a \omega) \\
& + \chi^{\alpha*} (\partial_\beta \chi_\alpha C^\beta + \chi_\beta \partial_\alpha C^\beta + \frac{1}{2} \rho_5 \chi_\alpha C_L + \frac{1}{4} \chi_\alpha C_W + \frac{i}{4} \rho_\alpha \eta_W + \nabla_\alpha \omega) \\
& + C_\alpha^* (\partial_\beta C^\alpha C^\beta - i \bar{\omega} \rho^\alpha \omega) \\
& + C_L^* (\partial_\alpha C_L C^\alpha + \frac{1}{2} \bar{\omega} \rho_5 \eta_W - \bar{\omega}_\alpha i \bar{\omega} \rho^\alpha \omega) \\
& + \omega^* (\partial_\alpha \omega C^\alpha + \frac{1}{2} \rho_5 \omega C_L + \frac{1}{4} \omega C_W + \chi_\alpha (i \bar{\omega} \rho^\alpha \omega)) \\
& + C_W^* (\partial_\alpha C_W C^\alpha - \bar{\omega} \eta_W) \\
& + \eta_W^* (\partial_\alpha \eta_W C^\alpha + \frac{1}{2} \rho_5 \eta_W C_L - \frac{1}{4} \eta_W C_W - i \rho^\alpha \omega (\partial_\alpha C_W + \bar{\chi}_\alpha \eta_W) - \frac{4}{e} \omega (\bar{\omega} \rho_5 T)) \\
& - \frac{1}{2e} (\psi^* \omega) (\psi^* \omega), \tag{7}
\end{aligned}$$

where C^α , ω , C_L , C_W and η_W are ghosts for reparametrization, local supersymmetry, local Lorentz rotation, Weyl and super-Weyl transformations respectively. T is defined as $T \equiv \epsilon^{\alpha\beta} \nabla_\alpha \chi_\beta$.

The BRST transformation in the space of fields and antifields is generated by S ,

$$\delta \cdot = (\cdot , S). \tag{8}$$

It is nilpotent *off-shell* as the result of the CME (6). Note that the solution S in (7) contains a quadratic term of anti-field ψ^* and the BRST transformation of the fermionic matter ψ depends on the antifield reflecting the properties of the on-shell algebra (4).

It is also useful to consider the following BRST operator in the classical basis. It acts on functional depending only on the fields as

$$\delta_0 \cdot = (\cdot , S)|_{\Phi^*=0}. \tag{9}$$

It is nilpotent *on-shell* of the classical equations of motion of the fermionic matter ψ ,

$$\delta_0^2 \psi = \frac{\omega}{e} (S_{0,\psi} \omega). \tag{10}$$

3 The Super Liouville action

Here we will construct the WZ term for the spinning string in terms of the anomalies of the theory. The antifield independent part of the anomaly in the diffeomorphism invariant regularizations is known as the super-Weyl anomaly. It has been given explicitly in a Hamiltonian BRST procedure [16]

$$\mathcal{A} = -i\hbar k \int d^2x [2 (\epsilon^{\alpha\beta} \partial_\alpha \bar{\omega}_\beta) C_W - 4 \bar{T} \rho_5 \eta_W] \equiv \mathcal{A}_a(\phi) c^a, \tag{11}$$

where k is a constant proportional to $(10 - D)$ and c^a denote ghosts for the anomalous transformations; *i.e.* the Weyl and super-Weyl ghosts, C_W and η_W . The super-Weyl anomaly (11) verifies

$$\delta_0 \mathcal{A} = 0. \quad (12)$$

The cohomological reconstruction procedure [5][6] allows us to reconstruct a BRST invariant object depending on fields and antifields from a weakly δ_0 invariant antifield independent object. Since (12) holds off-shell we can conclude that \mathcal{A} is actually the complete anomaly and does not depend on the antifields. This fact can be understood intuitively by taking into account the fact that the algebra is open only for the fermionic matter ψ while the super-Weyl anomaly \mathcal{A} does not contain ψ . The corresponding WZ term $\mathcal{M}_1(\phi, \theta^a)$ will depend only on the fields and verifies

$$\delta_0 \mathcal{M}_1(\phi, \theta^a) = i \mathcal{A}_a(\phi) c^a, \quad (13)$$

where θ^a indicates parameters of the anomalous subgroup. This antifield independent part of the WZ term becomes actually the full WZ term satisfying

$$\delta \mathcal{M}_1(\phi, \theta^a) = i \mathcal{A}_a(\phi) c^a \quad (14)$$

in this case.

The WZ term can be obtained in terms of the anomaly as

$$\mathcal{M}_1(\phi, \theta) = -i \int_0^1 dt \mathcal{A}_a(F(\phi, t\theta)) \lambda_b^a(t\theta, \phi) \theta^b. \quad (15)$$

Here $F^i(\phi, \theta^a)$ are the finite anomalous transformations of ϕ^i and satisfy the on-shell composition law

$$F^i(F(\phi, \theta), \theta') = F^i(\phi, \varphi(\theta, \theta'; \phi)) + M^{ij}(\theta, \theta'; \phi) \mathcal{S}_{0,j}(\phi), \quad (16)$$

where $M^{ij}(\theta, \theta'; \phi)$ represents the open structure at finite level [1]. λ_b^a is an inverse of μ_b^a defined by using the composition function of the quasi-group as;

$$\mu_b^a(\theta, \phi) = \left. \frac{\partial \varphi^a(\theta, \theta', \phi)}{\partial \theta'^b} \right|_{\theta'=0}. \quad (17)$$

For the spinning string these transformations are

$$\tilde{X}^\mu = X^\mu, \quad \tilde{\psi}^\mu = e^{\frac{\sigma}{4}} \psi^\mu, \quad \tilde{e}_\alpha^a = e^{\frac{\sigma}{2}} e_\alpha^a, \quad \tilde{\chi}_\alpha = e^{\frac{\sigma}{4}} (\chi_\alpha + \frac{i}{4} \rho_\alpha \eta), \quad (18)$$

where σ and η are, respectively, the Weyl and the super Weyl finite transformation parameters. The composition law of two finite anomalous transformations is expressed by

$$\varphi^\sigma(\theta, \theta') = \sigma + \sigma', \quad \varphi^\eta(\theta, \theta') = \eta + e^{\frac{\sigma}{4}} \eta' \quad (19)$$

and the λ^a_b is given by $\begin{pmatrix} 1 & 0 \\ 0 & e^{-\frac{\pi}{4}} \end{pmatrix}$.

The WZ term is constructed using (15) as,

$$\mathcal{M}_1 = \hbar k S^{SL}, \quad (20)$$

where

$$S^{SL} \equiv S_0^{SL} + S_1^{SL}, \quad (21)$$

$$S_0^{SL} = - \int d^2x \, e \left\{ \frac{1}{2} (g^{\alpha\beta} \partial_\alpha \sigma \partial_\beta \sigma - i \bar{\eta} \rho^\alpha \nabla_\alpha \eta) - \bar{\chi}_\alpha \rho^\beta \rho^\alpha \eta \partial_\beta \sigma + \frac{1}{4} \bar{\eta} \eta \bar{\chi}_\alpha \rho^\beta \rho^\alpha \chi_\beta \right\}, \quad (22)$$

$$S_1^{SL} = - \int d^2x [2 \epsilon^{\alpha\beta} \partial_\alpha \varpi_\beta \sigma - 4 \bar{T} \rho_5 \eta]. \quad (23)$$

It is the super-Liouville action found in [17][18]. Remember S_0^{SL} is obtained from the classical action by the replacements $X \rightarrow \sigma$ and $\psi \rightarrow -\eta$ and S_1^{SL} is obtained from the anomaly by the replacements $C_W \rightarrow \sigma$ and $\eta_W \rightarrow \eta$.

The transformation properties of the extra variables are determined by requirements that $F^i(\phi^j, \theta^a)$'s in (18) are invariant under the anomalous transformations and covariant under non-anomalous transformations. They are

$$\begin{aligned} \delta_0 \sigma &= \partial_\alpha \sigma \, \epsilon^\alpha - \bar{\eta} \epsilon - \epsilon_W \\ \delta_0 \eta &= \partial_\alpha \eta \epsilon^\alpha + \frac{1}{2} \rho_5 \eta \epsilon_L + i \rho^\alpha \epsilon (\partial_\alpha \sigma + \bar{\eta} \chi_\alpha) - \frac{1}{4} \eta \epsilon_W - \epsilon_{SW}. \end{aligned} \quad (24)$$

The super-Liouville action is invariant under non-anomalous transformations ; reparametrization (ϵ^α), local Lorentz (ϵ_L) and SUSY transformation (ϵ). Under anomalous transformations ; Weyl (ϵ_W) and super Weyl (ϵ_{SW}), S^{SL} is not invariant. It is the source of anomaly cancellation.

4 On-shell structure of the extended formalism

Now we want to study the BRST transformations of the extra variables in more detail. Let us now construct the extended action in the space of fields and antifields. We start considering a classical action that includes the transformation properties of the Liouville and super-Liouville fields as well as the original ones[3][4],

$$\begin{aligned} S^{(ext)} &= S + \sigma^* (\partial_\alpha \sigma \, C^\alpha - \bar{\eta} \omega - C_W) \\ &\quad + \eta^* (\partial_\alpha \eta \, C^\alpha + \frac{1}{2} \rho_5 \eta \, C_L - \frac{1}{4} \eta \, C_W + i \rho^\alpha \omega (\partial_\alpha \sigma + \bar{\eta} \chi_\alpha) - \eta_W). \end{aligned} \quad (25)$$

$S^{(ext)}$ does not verify the classical master equation because the algebra of the transformations of the extra variables (24) is not closed, or equivalently its BRST transformations do not satisfy the nilpotency,

$$\delta_0^2 \eta = - (\bar{\omega} a) \, \omega, \quad a \equiv i \rho^\alpha \nabla_\alpha \eta + \rho^\alpha \rho^\beta \chi_\alpha (\partial_\beta \sigma + \bar{\eta} \chi_\beta) - \frac{4}{e} \rho_5 T. \quad (26)$$

Note a is the Euler derivative of the super Liouville action with respect to η .

The situation is similar to the transformation of the fermionic matter ψ in which the closure of the algebra requires the classical equation of motion of ψ . However we don't have the classical equation of motion for the extra variable η . The infinitesimal algebra closes and has nilpotent BRST transformation when we take the WZ term as is describing the dynamical action of the super Liouville fields. In fact the WZ term enters in the quantum action. It has the coefficient proportional to \hbar while $\theta^* \delta_0 \theta$ term does not contain \hbar . In addition bilinear terms of antifields reflecting the open algebra structure of the extra variables, symbolically expressed as $(\theta^*)^2$, are proportional to \hbar^{-1} ,

$$W = S + \hbar S^{WZ} + \theta^* \delta_0 \theta + \frac{1}{\hbar} (\theta^*)^2 + \dots \quad (27)$$

In case of absence of $(\theta^*)^2$ term we can take the classical limit $\hbar \rightarrow 0$ to find $S^{(ext)}$. In presence of $(\theta^*)^2$ term we will take the classical limit only after making a canonical transformation [19] [6] [4]

$$\theta = \hbar^{-1/2} \tilde{\theta}, \quad \theta^* = \hbar^{1/2} \tilde{\theta}^*. \quad (28)$$

After this transformation

$$W = W_0 + \hbar^{1/2} M_{1/2} + \dots \quad (29)$$

The classical limit gives W_0 and is the extended proper solution of CME. The second term $M_{1/2}$ is the first order quantum correction referred as the background term [4].

In the present model W is

$$W = S + \hbar k S^{SL} + \sigma^* \delta_0 \sigma + \eta^* \delta_0 \eta - \frac{1}{2\hbar k e} (\eta^* \omega)(\eta^* \omega). \quad (30)$$

It satisfies

$$(W, W) = 2i \mathcal{A}. \quad (31)$$

After the canonical transformation (28) we have that W_0 and $M_{1/2}$ are given by

$$\begin{aligned} W_0 &= S + k S_0^{SL} + \sigma^* (\partial_\alpha \sigma C^\alpha - \bar{\eta} \omega) \\ &\quad + \eta^* (\partial_\alpha \eta C^\alpha + \frac{1}{2} \rho_5 \eta C_L - \frac{1}{4} \eta C_W + i \rho^\alpha \omega (\partial_\alpha \sigma + \bar{\eta} \chi_\alpha)) - \frac{1}{2k e} (\eta^* \omega)^2, \end{aligned} \quad (32)$$

$$M_{1/2} = -k [2\epsilon^{\alpha\beta} \partial_\alpha \varpi_\beta \sigma - 4 \bar{T} \rho_5 \eta] - \sigma^* C_W - \eta^* \eta_W. \quad (33)$$

σ and η in the above expressions are understood to be new variables $\tilde{\sigma}$ and $\tilde{\eta}$. They satisfy

$$(W_0, W_0) = 0, \quad (W_0, M_{1/2}) = 0, \quad (34)$$

$$\hbar (M_{1/2}, M_{1/2}) = 2\hbar k [2\epsilon^{\alpha\beta} \partial_\alpha \varpi_\beta C_W - 4 \bar{T} \rho_5 \eta_W] = 2i \mathcal{A}. \quad (35)$$

W_0 should be regarded as an extended classical action. The first two equations mean that both W_0 and $M_{1/2}$ are invariant under BRST transformation generated by W_0 . The last expression has the form of the super-Weyl anomaly.

In W_0 the extra variables σ and η are coupled with super-gravity multiplet exactly in the same way as X^μ and ψ^μ . W_0 is expressed in the same form as S in (6) if (σ, η) is regarded as $(D+1)$ -th component of (X^μ, ψ^μ) . The Liouville and super-Liouville fields give additional contribution to the anomaly and shift the proportional constant $(10 - D)$ in the coefficient k of the anomaly to $(10 - (D + 1)) = (9 - D)$. After this replacement the background term will cancel the anomaly and the quantum master equation is satisfied at one-loop.

5 Summary and discussion

We have constructed the extended field-antifield formalism of the spinning string as an example of an anomalous theory with a open and irreducible gauge algebra. We have followed the general discussion and constructed the WZ term based on the super-Weyl anomaly via the finite transformations[1].

A new phenomena appears at classical level since there is no solution of the classical master equation in the extended space. The nilpotency of BRST transformation for the fermionic Liouville field η is not satisfied. The non-vanishing term is actually proportional to the dynamical equation of motion for the fermionic Liouville field obtained from the WZ term, namely super-Liouville action. The extended solution including the super-Liouville action is not regarded as a classical object. In this situation quadratic terms of antifield of fermionic Liouville field appears in the quantum action and are proportional to \hbar^{-1} . In order to take the classical limit $\hbar \rightarrow 0$, one need to perform the canonical transformation (28). After the canonical transformation is performed we can split the extended action into an action W_0 verifying the classical master equation and the background term $M_{1/2}$ which is proportional to $\hbar^{\frac{1}{2}}$. W_0 defines an off-shell nilpotent BRST transformation and the background term cancels the anomaly at one-loop level.

The origin of the non-nilpotency of the extra variables is the fact that commutator of two non-anomalous (SUSY) transformations do not close but gives anomalous (super-Weyl) transformations. This phenomena generally occurs when a part of gauge symmetries is broken due to anomalies but the non-anomalous transformations do not form a sub-algebra. The analysis of this phenomena in a generic gauge theory will be discussed elsewhere [20].

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